Government of **Western Australia**
School Curriculum and Standards Authority

WACE
Western Australian Certificate of Education

Western Australian Certificate of Education Examination, 2014

Question/Answer Booklet

MATHEMATICS: SPECIALIST 3C/3D

Section One: Calculator-free

Place one of your candidate identification labels in this box.

Ensure the label is straight and within the lines of this box.

Student Number: In figures | | | | | | | | | |

In words _____

_____

Number of additional answer booklets used (if applicable):

Time allowed for this section

Reading time before commencing work: five minutes
Working time for this section: fifty minutes

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer Booklet
Formula Sheet

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of exam
Section One: Calculator-free	8	8	50	50	33⅓
Section Two: Calculator-assumed	12	12	100	100	66⅔
				Total	100

Instructions to candidates

1. The rules for the conduct of Western Australian external examinations are detailed in the *Year 12 Information Handbook 2014*. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer Booklet.

3. You must be careful to confine your responses to the specific questions asked and to follow any instructions that are specific to a particular question.

4. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
   - Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
   - Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.

5. **Show all your working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

6. It is recommended that you **do not use pencil**, except in diagrams.

7. The Formula Sheet is **not** to be handed in with your Question/Answer Booklet.

See next page

Section One: Calculator-free **(50 Marks)**

This section has **eight (8)** questions. Answer **all** questions. Write your answers in the spaces provided.

Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.

- Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
- Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.

Working time: 50 minutes.

Question 1 **(5 marks)**

An apple grower can sell 160 bags of apples at a profit of $3 per bag.

For each week he delays his delivery of the apples, he can sell an extra 80 bags, but his profit will be reduced by 50 cents per bag.

After how many weeks should the apple grower send his apples to market if he is to maximise his profit?

Question 2 **(6 marks)**

(a) Let x be an angle measured in radians.
Show by first principles that $\frac{d}{dx}(\sin x) = \cos x$, by considering

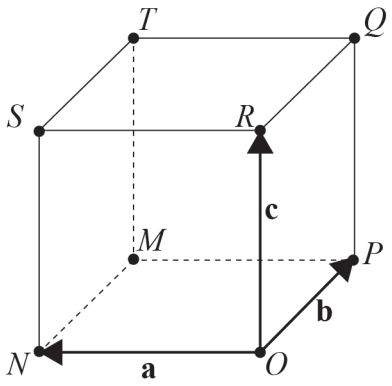
$$\frac{d}{dx}(\sin x) = \lim_{h \to 0} \frac{\sin(x + h) - \sin x}{h}.$$ (4 marks)

(b) Let $\theta°$ be an angle measured in degrees.
Use the chain rule to determine $\frac{d}{d\theta}(\sin \theta°)$ in terms of $\theta°$. (2 marks)

See next page

Question 3 **(5 marks)**

Let $MNOPQRST$ be a rectangular prism with sides $\overrightarrow{ON}$, $\overrightarrow{OP}$ and $\overrightarrow{OR}$ denoted by vectors $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ respectively, as shown in the diagram below.



Suppose that A is the midpoint of $\overrightarrow{MN}$, B is the midpoint of $\overrightarrow{MT}$, C is the midpoint of $\overrightarrow{QR}$ and D is the midpoint of $\overrightarrow{OR}$.

(a) Express $\overrightarrow{OA}$, $\overrightarrow{OB}$, $\overrightarrow{OC}$ and $\overrightarrow{OD}$ in terms of $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$. (2 marks)

(b) Prove that the quadrilateral $ABCD$ is a parallelogram. (3 marks)

Question 4 **(10 marks)**

(a) Determine expressions for the following indefinite integrals.

(i) $\int \frac{\cos(\ln(x))}{x}\, dx$ (3 marks)

(ii) $\int \cos^3 x\, dx$ (3 marks)

(b) Use the substitution $u = 2x - 1$ to determine $\int_0^1 x\,(2x-1)^{1006}\,dx$. (4 marks)

See next page

Question 5 **(5 marks)**

The tangent to the curve $y^2 = x^3$ at the point $P(1,1)$ meets the x-axis at Q and the y-axis at R.

Determine the ratio $PQ : QR$.

See next page

Question 6 **(7 marks)**

(a) Show that $(1+i)^5 = -4-4i$. (3 marks)

(b) Hence determine all the roots of the equation $z^5 = -4-4i$, expressing each in the form $r\,\text{cis}\theta$ with $r \geq 0$ and $-180° < \theta \leq 180°$. (4 marks)

See next page

Question 7 **(8 marks)**

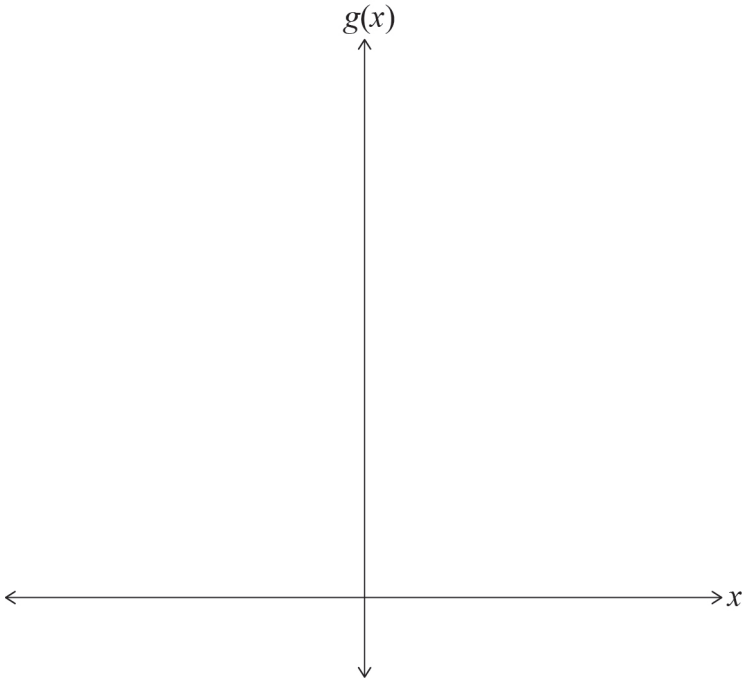
Consider the function $f(x) = \dfrac{100}{1 + e^{-x}}$ defined for all real numbers, x.

(a) Determine $f'(x)$. (2 marks)

(b) Since $f(x)$ can be written alternatively as $f(x) = \dfrac{100\ e^x}{e^x + 1}$, hence or otherwise determine $\int f(x)dx$. (3 marks)

See next page

(c) By considering the value of $g(0)$ and limiting values as $x \to \pm\infty$, sketch the graph of $g(x) = \dfrac{100}{1 + e^{-|x|}}$ on the axes below. (3 marks)



Question 8 **(4 marks)**

In the branch of mathematics known as number theory, one important family of equations are the *Diophantine* equations.

Consider the Diophantine equation

$$x^2 - 9y^2 = 11.$$

Solutions are required in which x and y are both positive integers.

By factorising, or otherwise, use proof by contradiction to show that the equation cannot have any positive integer solutions.

End of questions

Additional working space

Question number: _____

Additional working space

Question number: _____

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